

RPN Tutorial, incl. some [things HP did not tell](#)

Copyright © 2014 by Hans Klaver, The Netherlands

RPN, *postfix notation* or *stack logic*, the calculator logic system used in many Hewlett-Packard (HP) calculators and simulations thereof (like the standard [Calculator](#) in Mac OS X in RPN-mode, start with \mathbb{R} or Command+R), is easy to use and saves you time because there is no need to use brackets or =. What's better, it gives you more insight into your calculations than using the 'algebraic' systems used by other calculators and it keeps *you* and not your calculator (see [Appendix D](#)) in command of what is calculated.

In 1972, more than 40 years ago, HP launched the [HP-35](#), the first pocket calculator with transcendental functions and the first with RPN. This same RPN is used in calculators sold by HP nowadays, like the HP-12C, HP-12C Platinum, HP-15C LE, HP 17bII+ and HP 35s, also in the amazing [WP 34S](#) and [WP 31S](#), and in a slightly modified version in the HP 20b and 30b and in graphing calculators like the HP 50g and HP Prime. This tutorial is a tribute to the HP-35 and especially to the lay-out of the beautiful and informative [HP-35 Operating Manual](#). As far as possible it uses the colours of the keys as they appear in that Manual, so the colours of your calculator's keys will almost always differ; don't get confused by that.

The 'Cheat Sheet' below serves as a table of contents and as a summary for this tutorial.

You can learn the basics of RPN (1., 2., 3. & 4. of the Cheat Sheet) in less than 15 minutes. The finer details (5., 6. & 7.) take another 30 minutes to get used to. Understanding of 8. is a bit more difficult and probably is unnecessary, but if you are a real 'formula cruncher' study of that point will be well paid off.

The [Appendices](#) elaborate on a few interesting points.









How to practice?

To get a feeling for the HP-35 you could use Neil Fraser's JavaScript [HP-35 simulator](#). With this simulator you can only use a pointing device (e.g. mouse) to press keys.

If you don't have a hardware RPN calculator use your computer keyboard and practice on the [famous HP-15C](#) with the online version of Greg Hewgill's [HP-15C simulator](#). With this simulator you can watch the stack live! Open in a separate window on Mac OS X by simultaneously pressing \mathbb{C} (option+command) while clicking the link; or on Windows, Mac OS or Linux right-click the link and choose 'Open in a separate window'.

Or download one of the numerous RPN calculator programs for your computer or smartphone. E.g. the HP Prime simulator for Windows (<ftp.hp.com/pub/calculators/Prime/>) or the [WP 34S emulator](#) for Windows, Mac OS X, Linux or [iOS](#).

RPN Cheat Sheet

- One-number calculations** (such as $1/x$, \sqrt{x} , x^2 , $\cos x$, $\log x$): first key-in the number, then press the function key (or press the applicable prefix ('shift') key, then the function key).
Example: calculate $\sqrt{2}$ Press:   Result: **1.41421356237**
Example: calculate 34^2 Press:   Result: **1156**
- Two-number calculations** (+, -, \times , \div , y^x , %, $\Delta\%$, %T): key-in the first number, press ENTER to separate the first number from the second, key-in the second number (do *not* press ENTER after the second number!)*, then press the function key.
Example: calculate $12 + 3$ Press:     Result: **15**
- Series of additions and/or subtractions** (like $12 + 34 + 56 - 78 + 90 - 12$): ENTER should only be used to separate two numbers keyed-in without a function key in-between (in this case between the first and the second number), all other numbers are separated by the function keys (which also save the intermediate results).

12 ENTER↑ 34 + 56 + 78 - 90 + 12 - Result: 102

There is *no limit* to the number of additions and/or subtractions that can be done in a row.

4. **Chain calculations**, for example $(12 \times 34) + (56 \times 78) - (90 \times 12)$: work them as on paper, like so

12 ENTER↑ 34 ×
56 ENTER↑ 78 × +
90 ENTER↑ 12 × - Result: 3696

There is *no limit* to the number of operations that can be done in a row when there is only *one level of parentheses*, provided that all parentheses have been placed correctly to indicate the order of precedence of all operations.

5. **Calculations with nested parentheses** like $3 \times (4 + (5 \times (6 + 7)))$: begin with the *innermost set of parentheses* and work outwards. With this technique nearly every complicated calculation can be done on a 4-level stack RPN-calculator.

6 ENTER↑ 7 + 5 × 4 + 3 × Result: 207

Calculations like the one above (in which more than 4 numbers have to be keyed-in before the first operation key is pressed) cannot be done in a pure left-to-right way on a 4-level stack calculator.

6. **Use of $x \leftrightarrow y$ or \blacktriangleright or \blacktriangleright in nested calculations**. To get the order of the numbers right when you have to do a *non-commutative* operation ($-$, \div , y^x) 'backwards', just before pressing the operation key press $x \leftrightarrow y$ ('exchange x and y'). In the HP 20b/30b this is the $\frac{y}{x}$ key. In HP graphing calculators use SWAP: before the operation key press ENTER followed by \blacktriangleright or \blacktriangleright ($\frac{y}{x}$).

E.g.: $3 \times (4 - (5 \times (6 + 7)))$

6 ENTER↑ 7 + 5 × 4 $x \leftrightarrow y$ - 3 × Result: -183

Beware of the order of operations when there are several additions and subtractions or multiplications and divisions next to each other! (see [explanation](#)).

7. **Use of STO and RCL**. If you're not sure that the 4-level stack will be enough then use additional memory with **STO** and **RCL**. When there is a non-commutative operation ($-$, \div , y^x) somewhere in the middle of the formula, start with the part *after* this operation, STOr the intermediate result, and when the part *before* this operation is done, RCL (recall) the intermediate result and press the middle operation key. In this way you get the order of the numbers right without using $x \leftrightarrow y$, so you save one keystroke.

E.g.: $(2^9 + 3^8)(4^7 + 5^6) - (6^5 + 7^4)(8^3 + 9^2)$

First think what this actually means:

$[(2^9 + 3^8) \times (4^7 + 5^6)] - [(6^5 + 7^4) \times (8^3 + 9^2)]$

There are implied parentheses around each term of this calculation and that's why this isn't an example with only one level of parentheses (see 4.), so you aren't sure whether the 4-level stack will be enough for this calculation (actually it isn't). The easiest way to solve this one is by starting after the minus - :

6 ENTER↑ 5 y^x 7 ENTER↑ 4 y^x +
8 ENTER↑ 3 y^x 9 ENTER↑ 2 y^x + × STO↑
2 ENTER↑ 9 y^x 3 ENTER↑ 8 y^x +
4 ENTER↑ 3 y^x 5 ENTER↑ 6 y^x + × RCL - Result: 220,364,696

8. **How to recognize and handle calculations that cannot be done using a 4-level stack?**

- Obviously the 4-level stack has only four storage locations, holding the number in the display and three additional numbers. These four numbers can either be newly keyed-in numbers, or intermediate results from calculations or any combination thereof, but *the total may never exceed four!*
- When you use ENTER↑ once to key-in two new numbers only *two* intermediate results from previous operations will be preserved.

Keeping these two rules in mind a **simple counting technique** suffices to recognize calculations that cannot be done using only the 4-level stack. When you always start calculations from within the innermost set of parentheses such situations will rarely arise. But when they do arise you can always handle them using additional memory with **STO** and **RCL**.

* If you do press ENTER after the second number you get wrong results on classical RPN calculators! On the recent models HP 20b and 30b and on the graphing HP calculators like the series 28, 48 & 49 and the 50g and Prime (which use a variant of RPN called [Entry RPN or RPL](#)) it is allowed to press ENTER after the second number, but it is not necessary. See [Appendix A. The Stack](#) and [Appendix G. RPN Variants](#).

† The HP-35 had only one additional memory location apart from the stack; later models need a register number or letter after **STO** and **RCL**, e.g.: **STO** **1** and **RCL** **1**.

Basics of RPN

1. Principles. RPN is based upon four leading principles:

- The number or two numbers necessary for an operation (calculation) should be in the calculator before an operation key (operator) is pressed. So the order is always: *first number(s), then operator*.
- The *result* of every operation immediately appears in the display and *overwrites* the original number or two numbers within the calculator.
- The *result* of every operation can be used by a *next* operation and is stored automatically when a new number is keyed-in.
- Only when *two* new numbers have to be keyed-in before the next operation can be done, the two numbers should be separated from each other by a special key: **ENTER**↑.

2. One number calculations. The order of entering numbers and operations in RPN can easiest be seen with single number functions. No matter if you write the math operator in front of the number (like in \sqrt{x} or $\log x$ or $1/x$) or behind the number (like in x^2 or $x!$), when using an RPN calculator the order of calculations is *always*: first number, then operator.

Try the examples below. It is *not* necessary to clear the number in the display ('x') between two different calculations, because numbers present in the calculator as a result of a previous operation are 'pushed upwards' (and can be used later in a calculation if necessary). If you would make an entry mistake, you can use **CL x** (Clear x) or **←** (Backspace) and retype the number correctly.

Example I: $\sqrt{169}$ (the square root of 169)

Press	Display	Comments
169	169	First key-in the number (argument of the function); do <i>not</i> use ENTER!
√x	13	Press the √x button and the answer appears right away (there's no = button)

Example II: $9!$ (the factorial of $9 = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9$)

Press	Display	Comments
9	9	Key-in the number (argument of the function); don't press ENTER
← x!	362,880	For some functions first press a prefix ('shift') key *

* The HP-35 had one prefix key: **ARC**. Most later models have several. They have interesting nicknames: **←** 'gold', **f** 'fold', **f¹** 'dolf', **←** 'blue', **g** 'glue', **h** 'hack'. Also **HYP** and **HYP¹** are (shifted) prefix keys.

One-number operations always act upon the number in the display, called x , and the result of the operation overwrites this number and becomes the new x . This new x can be used as the argument of another one-number operation (or as one of the two arguments of a two-number operation).

Example III: $\frac{1}{\sqrt{121}}$

Press	Display	Comments
121	121	Because the calculator has 1/x as a separate function we start with $\sqrt{121}$
√x	11	Now the result of $\sqrt{121}$ can be used as an argument for 1/x
1/x	0.0909090901	The (rounded) result of $1/11$

Press	Display	Comments
-------	---------	----------

For an explanation of the workings of the stack in one number calculations see [here](#) in Appendix A.

3. Two-number calculations.

RPN treats problems the way you do on paper.
For example, you would solve $456 + 123$ like this:

$$\begin{array}{r} 456 \\ 123 \\ \hline + \\ \hline 579 \end{array}$$

The sequence is: 456 123 +
This is the same order followed when using RPN calculators: key-in the first number, 456, then press **ENTER** to tell the calculator that you have completed the first number and that you want to enter a second number, then key-in the second number, 123, and press **+** to add both numbers. The answer, **579**, immediately appears in the display when you press **+**; both original numbers disappear from the calculator and are replaced by the result.

The same is true for other operations on two numbers like $-$ (subtraction), \times (multiplication), \div (division), and y^x (exponentiation or ‘power’ function: the number keyed-in first (y) raised to the power of the second number (x)). Try it!

456	456	456	
123	123	$\frac{456}{123} = 3.707317073$	$3^{-10} = 0.00001693508781$
$\frac{456}{123} -$	$\frac{456}{123} \times$		
333	56088		

Example IV: $456 - 123$

Press	Display	Comments
456	456	The number keyed-in appears in the display
ENTER	456	ENTER separates the first number from the second
123	123	Again the keyed-in number is seen in the display
-	333	Press the - button and the answer appears right away (there’s no = button)

Example V: 456×123

456 **ENTER** 123 **x** Answer: **56,088**

Example VI: $456/123$

456 **ENTER** 123 **÷** Answer: **3.70731707317**

Example VII: 3^{-10}

Press	Display	Comments
3	3	First key-in the <i>base</i> number, which is designated by the y in y^x
ENTER	2	ENTER separates the <i>base</i> from the second number (the <i>exponent</i>)
10	10	Key-in the <i>exponent</i> , which is designated by the x in y^x
		C hange the S ign of the exponent, making it negative

Press	Display	Comments
CH S *	-10	
y^x †	0.00001693508781	Press the y^x button to calculate the power

* On some models **+/-**

† The HP-35 is the only early model with a **x^y** key, and there the exponent is keyed-in first. Unlike all other models the HP Prime has a key labelled **x²**, which operates like the **y^x** key in the example.

Also percentage operations (% , Δ% and %T) are two-number calculations.

Example VIII: What is 21% of 367 billion?

Press	Display	Comments
367 E EX 9	367 09	Enter 9 as EX ponent of ten*, so now we have 367×10^9
ENTER †	3.670,000 11	$367 \times 10^9 = 3.67 \times 10^{11}$
21	21	The percentage
%	7.707,000 10	$21\% \text{ of } 367,000,000,000 = 7.707,000 \times 10^{10} = 77,070,000,000 = 77.07 \text{ billion}$

* Exact powers of ten are easy to key-in, e.g. 1 million = 10^6 : **E EX** 6 and $0.0001 = 10^{-4}$: **E EX** 4 **CH S**

See [Appendix C](#) for more examples of percentage calculations.

For an explanation of the workings of the stack in two number calculations see [here](#) in Appendix A.

4. Serial calculations like $12 + 34 + 56 - 78 + 90 - 12$.

Another example that the result of an operation is itself used by the next operation is the calculation of the square root of an addition: $\sqrt{(12 + 34)}$. First do the addition: 12 **ENTER** † 34 **+** and when the intermediate result 46 is in the display press the **√x** button to perform the square root function. Result: 6.782329983

In the same way the result of an addition can be used by another addition, and the next result by yet another addition, etc. To add the three numbers 12, 34 and 56 you simply press: 12 **ENTER** † 34 **+** 56 **+**. Result: 102. Next you can do a subsequent addition by keying in the number and press **+**, or a subsequent subtraction by keying in a number followed by **-**. There is *no limit* to the number of additions or subtractions you can do in one row.

Example IX: $12 + 34 + 56 - 78 + 90 - 12$

Press	Display	Comments
12 ENTER †	12	ENTER † separates the first number from the second
34 +	46	The following numbers are separated by the operation key
56 +	102	Each intermediate result can be inspected in the display
78 -	24	
90 +	114	
12 -	102	The final result

The **ENTER↑** key is used to separate two numbers entered immediately after each other, such as the first two numbers; all following numbers are separated by the operation keys (in this case **+** or **-**), which also automatically save the calculated (or *intermediate*) results.

4. Chain calculations

Now look at this problem with parentheses:

$(56 \times 78) - (12 \times 34)$

If you solved it on paper, you would first multiply 56×78 , then 12×34 , then subtract the two intermediate results to get the final result. That's just the way you do it using an RPN-calculator, without the need to key-in the parentheses. In this calculation you must use **ENTER↑** two times since you must key-in another pair of numbers before you can perform a function.

Example X: $(56 \times 78) - (12 \times 34)$

Press	Display	Comments
56 ENTER↑	56	ENTER↑ separates the first number from the second
78 ×	4368	The result of (56×78) , saved automatically in the next step
12 ENTER↑	12	Now we start within the second pair of parentheses
34 ×	408	The result of (12×34)
-	3960	Finally we do the subtraction $4368 - 408$ to get the end result

Note that you only have to press **ENTER↑** to separate two numbers that are keyed-in without an operation key in-between. The intermediate results are automatically saved in the calculator for you! Just as you would work out the problem with paper and pencil, you solve what's within the parentheses first and subtract **-** last. The stack memory 'automagically' retrieves the intermediate results for the final subtraction operation.

Mixed one- and two-number operations follow the same principle: first key-in the number(s), then the operation(s); no parentheses, no 'equals' (=) key to press, the same order you would use on paper. For example:

Example XI: $(13 + 8) \times \log 100$

Press	Display	Comments
13 ENTER↑	13	ENTER↑ separates the first from the second number
8 +	21	This intermediate result is saved automatically
100	100	The argument for the log function is keyed-in
LOG	2	Calculate log 100
×	42	The final answer: 42

Another example of mixed one and two-number operations; again the intermediate results are saved automatically:

Example XII: $23^2 - (13 \times 9) + (5/7)$

Press	Display	Comments
23 □ x²	529	First, calculate 23^2
13 ENTER↑ 9 ×	117	then the product 13×9

Press	Display	Comments
−	412	subtract the two intermediate results
5 ENTER ↑ 7 ÷	0.714285714286	calculate 5/7
+	412.714285714	add the two last results to get the final result

There is *no limit* to the number of operations that can be done in a row when there is only *one level of parentheses*, provided that all parentheses have been placed correctly to indicate the order of preference of all operations.

Finer details of RPN

5. Nested calculations

Also calculations with nested parentheses are best worked the same way you would use on paper, in which case there is no other way than to *begin at the innermost set of parentheses, and work outwards*. Consider the following calculation:

$$3[4 + 5(6 + 7)]$$

First think what this actually means: $3 \times [4 + (5 \times (6 + 7))]$

Example XIII: $3 \times [4 + (5 \times (6 + 7))]$

Press	Display	Comments
6 ENTER ↑ 7 +	13	Intermediate result of 6 + 7
5 ×	65	Intermediate result of 5 × (6 + 7)
4 +	69	Intermediate result of 4 + (5 × (6 + 7))
3 ×	207	Final result: 3 × [4 + 5 × (6 + 7)]

Doing a calculation starting at the innermost set of parentheses and working outwards has several advantages:

- It takes fewer keystrokes than working from left to right.
- It is easier to keep track of what you are doing.
- It requires fewer registers on the stack. This is *very* important when using an RPN calculator with a *4-level stack* (all HP RPN calculators except the HP graphing calculators of the series 28, 48 and 49, the HP 50g and Prime; also the WP 31S and WP 34S in 8-level stack mode are an exception). Working the above calculation in a left-to-right order with a 4-level stack machine will overflow the 4-level stack and give a wrong result (without warning...). The above mentioned graphing calculators have a virtually *unlimited stack* and are immune to this problem.

Example XIV: $3 \times [4 + (5 \times (6 + 7))]$. Alternative method 1: pure left-to-right order.

This method takes more keystrokes than the method used in example XIII. It only gives a correct result when the RPN-stack has at least 5 levels; it gives a *wrong result* on 4-level stack RPN calculators due to stack overflow!

Press	Display	Comments
3 ENTER ↑	3	First number on the stack
4 ENTER ↑	4	Second number on the stack
5 ENTER ↑	5	Third number on the stack
6	6	Fourth number on the stack: now a 4-level stack is full!

Press	Display	Comments
ENTER↑	6	The first number on a 4-level stack (3) is lost!
7	7	
+	13	
×	65	
+	69	Intermediate result of $4 + (5 \times (6 + 7))$
×	207	Correct answer only when there are at least 5 stack levels *

* A 4-level stack RPN calculator will give the wrong answer 276 ($= 4 \times 69$).

For an explanation of the workings of the stack in nested calculations see [here](#) in Appendix A.

Example XV: $3 \times [4 + (5 \times (6 + 7))]$. Alternative method 2: partial left-to-right order.

This method gives a correct result on all RPN calculators, including 4-level stack models.

Press	Display	Comments
4 ENTER↑	4	First number on the stack
5 ENTER↑	5	Second number on the stack
6 ENTER↑	6	Third number on the stack
7	7	Fourth number on the stack: now a 4-level stack is full!
+	13	Intermediate result of $(6 + 7)$. The first number (4) on a 4-level stack is <i>not</i> lost!
×	65	Interm. result of $5 \times (6 + 7)$. Now there's one stack level free for a new number.
+	69	Result of $4 + (5 \times (6 + 7))$. Now there are two stack levels free for new numbers.
3 ×	207	Final result. Correct answer on all RPN calculators.

It is obvious that both left-to-right methods take more keystrokes than the preferred method, working from the innermost set of parentheses outwards, which in addition gives you a better overview of the intermediate results. And on a 4-level stack machine the left-to-right method of nested calculations can easily lead to stack overflow, giving wrong results without warning. *So use the left-to-right method for calculations with nested parentheses only when you know exactly what you are doing!* Below are some hints on how to recognize nested calculations that can and cannot safely be done in a left-to-right manner.

6. Use of $x \leftrightarrow y$ or ► or ⌘ in nested calculations.

When doing nested calculations from the innermost set of parentheses outwards you will often have to do a non-commutative operation ($-$, \div , y^x) 'backwards', so that the order of the numbers would be wrong. Getting the order of the numbers right can often best be done using the x exchange y key $x \leftrightarrow y$ (in graphing calculators called SWAP, accessed with the right arrow key ►; in the HP 20b, 30b and HP Prime the key has a symbol like this: ↔). This key exchanges the number in the display (called x) with the number in the second stack level (called y). In these cases $x \leftrightarrow y$ or ► or ⌘ should be pressed immediately before pressing the $-$, \div or y^x key. In most HP graphing calculators SWAP only works when there is no command line or (in the Prime) the entry line is empty, so you might have to press ENTER before ► or ⌘ (↔).

Example XVI: $3 \times [4 - (5 \times (6 + 7))]$

Press	Display	Comments
6 ENTER↑ 7 +	13	Calculates $6 + 7$

Press	Display	Comments
5 \times	65	Intermediate result of 5×13
4 $\mathbf{x\overline{y}}$	65	Puts 4 before 65 so subtraction will be correct*
$-$	-61	Intermediate result of $4 - 65$
3 \times	-183	Final result: 3×-61

* In graphing calculators you will often have to put the 4 on the stack with an extra ENTER before you can use the SWAP function by pressing \blacktriangleright or $\mathbf{x\overline{y}}$.

Example XVII:
$$\frac{4}{14 + (7 \times 3) - 2}$$

Press	Display	Comments
7 ENTER 3 \times	21	Calculates 7×3
14 $+$ 2 $-$	33	Calculates denominator $21 + 14 - 2$
4 $\mathbf{x\overline{y}}$	33	Puts 4 before ('above') 33 in preparation for division*
\div	0.1212121212	Calculates $4 \div 33$, the answer

* In graphing calculators you will often have to put the 4 on the stack with an extra ENTER before you can use the SWAP function by pressing \blacktriangleright or $\mathbf{x\overline{y}}$.

You could have done the above calculation by using $\mathbf{1/x}$ after \div or by changing the division into a *multiplication of the reciprocal* (multiplicative inverse) using $\mathbf{1/x}$ and \times . Likewise you could have done Example XVI by using $\mathbf{CH S}$ after $-$ or by changing the subtraction into an *addition of a negative number* (additive inverse) using $\mathbf{CH S}$ and $+$.

You even can do a power calculation 'backwards' by using inverse functions (exponential and natural logarithm) and the equivalence $y^x = e^{x \ln y}$. So 7^5 can be calculated with 7 ENTER 5 $\mathbf{y^x}$ or with 5 ENTER 7 \mathbf{LN} \times $\mathbf{e^x}$. But for most of us the latter method would be quite convoluted. Here too RPN offers a perfectly simple and easy to remember method:

7^5 'backwards' using $\mathbf{x\overline{y}}$: 5 ENTER 7 $\mathbf{x\overline{y}}$ $\mathbf{y^x}$ Answer: 16,807

Using $\mathbf{x\overline{y}}$ (or SWAP with \blacktriangleright or $\mathbf{x\overline{y}}$) is a straightforward method which works for all non-commutative operations, including $\mathbf{y^x}$.

But beware: using $\mathbf{x\overline{y}}$ is *not* the same as using $\mathbf{1/x}$ and \times or using $\mathbf{CH S}$ and $+$!

These latter methods are 'more universal' in that they can be used safely for 'backwards' calculation of a series of multiplications and divisions or additions and subtractions.

Using $\mathbf{x\overline{y}}$ you have to watch out carefully for the correct order of operations!

E.g.: $1 - 2 + 3 - [4 - (5 \times (6 + 7))]$

This would be the wrong way:

6 ENTER 7 $+$ 5 \times 4 $\mathbf{x\overline{y}}$ $-$ 3 $\mathbf{x\overline{y}}$ $-$ 2 $+$ 1 $\mathbf{x\overline{y}}$ $-$

Answer: -65 Wrong!

Because a series of additions and subtractions should always be done from left-to-right (see [Appendix E](#)) there are implied parentheses in this calculation:

$$1 - 2 + 3 - [4 - (5 \times (6 + 7))] =$$

$$(1 - 2 + 3) - [4 - (5 \times (6 + 7))]$$

So a correct way would be:

6 ENTER 7 $+$ 5 \times 4 $\mathbf{x\overline{y}}$ $-$ 1 ENTER 2 $-$ 3 $+$ $\mathbf{x\overline{y}}$ $-$

Answer: 63

Using **CH S** and **+** you can do everything 'backwards' because addition is a commutative operation:

$$1 - 2 + 3 - [4 - (5 \times (6 + 7))] =$$

$$1 + -2 + 3 + -[4 + -(5 \times (6 + 7))]$$

6 **ENTER↑** **7** **+** **5** **×** **CH S** **4** **+** **CH S** **3** **+** **2** **CH S** **+** **1** **+**

Answer: **63**

Example XVIII: 4^{3^2}

Press	Display	Comments
3 ENTER↑ 2 y^x	9	Calculates 3^2
4 x^zy	9	Otherwise you would calculate 9^4
y^x	262,144	Calculates 4^9 , the answer

Example XIX: 4^{3^2} (alternative solution, from left to right)

Press	Display	Comments
4 ENTER↑	4	
3 ENTER↑	3	
2	2	
y^x	9	
y^x	262,144	Calculates 4^9 , the answer

The next calculation can only be done 'backwards' on 4-level stack RPN calculators, because from left to right there are 6 numbers (1, 1.001, -6.2, -2, 3, π) before we can do any operation. So first we will work the expression in the first exponent, then the base expression.

Example XX: $1 - 1.001^{(-6.2 - 2^{3\pi})}$

Press	Display	Comments
6.2 CH S ENTER↑	-6.2	We start with all the numbers of the exponent
2 ENTER↑	2	
3 π	3.14159265359	Before and after π * we need no ENTER↑ †
× y^x	687.2913357	Calculates $2^{3\pi}$
-	-693.4913357	Calculates $-6.2 - 687.2913357$ (the exponent is complete)
1.001 x^zy y^x	0.500001180	Calculates $1.001^{-693.4913357}$
1 x^zy -	0.499998820	Final result: $1 - 0.500001180$

* The HP-35 is the only HP pocket calculator with a separate key for π , so no 'shift' needed.

† The Mac OS X Calculator *does* need ENTER before and after π and e , which may be considered a bug rather than a feature.

The following calculation can be done both ways (left-to-right and 'backwards'; explanation below under 7.):

Example XXI: $(e - 1)^{2/(\pi - \log \pi)}$ (left-to-right)

Press	Display	Comments
-------	---------	----------

Press	Display	Comments
1 e^x	2.71828182846	There is no key for e , so calculate e^1
1 $-$	1.71828182846	Now we have $e - 1$
2	2	Before and after π it is <i>not</i> necessary to press $\text{ENTER}\uparrow$ *
π $\text{ENTER}\uparrow$	3.14159265359	If you <i>do</i> press $\text{ENTER}\uparrow$ then n in x is copied to y and so we have n twice †
LOG	0.497149872694	Calculates $\log n$
$-$	2.6444427809	Calculates $n - \log n$
\div	0.756302996777	Calculates $2 / 2.6444427809$
y^x	1.50592241348	the final result $1.71828182846^{0.756302996777}$

* The Mac OS X Calculator *does* need ENTER before and after n and e , which may be considered a bug rather than a feature.

† For an explanation of this duplicating property of ENTER see [Appendix A. The Stack](#).

Example XXII: $(e - 1)^{2/(\pi - \log \pi)}$ (alternative solution: ‘backwards’)

Press	Display	Comments
2	2	Before and after π it is <i>not</i> necessary to press $\text{ENTER}\uparrow$ *
π $\text{ENTER}\uparrow$	3.14159265359	The $\text{ENTER}\uparrow$ copies n in x into y †
LOG	0.497149872694	Calculates $\log n$
$-$	2.6444427809	Now we have $n - \log n$
\div	0.756302996777	Calculates $2 / 2.6444427809$ (the exponent is complete)
1 e^x	2.71828182846	There is no key for e , so calculate e^1
1 $-$	1.71828182846	Now we have $e - 1$
\leftrightarrow	0.756302996777	Swaps <i>base</i> and <i>exponent</i> before power function
y^x	1.50592241348	the final result $1.71828182846^{0.756302996777}$

* The Mac OS X Calculator *does* need ENTER before and after n and e , which may be considered a bug rather than a feature.

† For an explanation of this duplicating property of ENTER see [Appendix A. The Stack](#).

7. Use of STO and RCL when the 4-level stack is not enough.

If you stick to the method of working calculations with nested parentheses from the innermost set of parentheses outwards then you will probably never encounter a calculation that cannot be done without overflowing a 4-level stack. Nevertheless these calculations *do* exist, and below it is explained how to recognize them. It is easy to handle them using STO and RCL , so if you’re not sure whether the 4-level stack will be enough don’t hesitate to make use of these keys.

The HP-35 had only one additional memory register apart from the stack, so pressing STO was enough to copy the number in the display to this register. In all later HP calculators STO should be followed by one or two numbers to address a specified memory register. Pressing RCL , if necessary followed by a register number, copies the contents of this register back to the display. It is *not* necessary to use ENTER before or after RCL.

Keep in mind that use of STO and RCL reverses the order of the operation. This isn’t important in commutative operations ($+$, \times), but it is important in non-commutative operations ($-$, \div , y^x).

E.g.: $45 - 12$

45 **ENTER↑** 12 **−** Answer: 33
 45 **STO** 12 **RCL** **x↔y** **−** Answer: 33
 12 **STO** 45 **RCL** **−** Answer: 33

So when you want to use **STO** and **RCL** in a long calculation start with the part after the subtraction or division to save one keystroke: **x↔y**.

Example XXIII: $\frac{(3^{\frac{2}{7}} + 4^{\frac{4}{9}})}{(7^{\frac{1}{4}} + 8^{\frac{3}{5}})}$ If you're not sure that this one can be done on a 4-level stack, use **STO** and **RCL**.

Press	Display	Comments
7 ENTER↑ 1 ENTER↑ 4	4	First do the denominator
÷	0.25	Result of 1/4
y^x	1.62657656169779	Result of 7 ^{0.25}
8 ENTER↑ 3 ENTER↑ 5	5	
÷	0.6	Result of 3/5
y^x	3.4822022531845	Result of 8 ^{0.6}
+	5.10877881488228	Result of 1.62657656169779 + 3.4822022531845
STO	5.10877881488228	(or STO [n] *) Store in a non-stack storage location
3 ENTER↑ 2 ENTER↑ 7	7	Now do the numerator
÷	0.28571428571429	Result of 2/7
y^x	1.36873810664221	Result of 3 ^{0.28571428571429}
4 ENTER↑ 4 ENTER↑ 9	9	
÷	0.44444444444444	Result of 4/9
y^x	1.85174942457457	Result of 4 ^{0.44444444444444}
+	3.22048753121678	Result of 1.36873810664221 + 1.85174942457457
RCL	5.10877881488228	(or RCL [n] *) Recall denominator
÷	0.63038304219303	Result of 3.22048753121678 ÷ 5.10877881488228

* The HP-35 had only one memory location apart from the stack; later models need a register number after **STO** and **RCL**.

Example XXIV: $\frac{(3^{\frac{2}{7}} + 4^{\frac{4}{9}})}{(7^{\frac{1}{4}} + 8^{\frac{3}{5}})}$ Actually this one *can* be done on a 4-level stack without using **STO** and **RCL** by doing the last term 'backwards'.

Press	Display	Comments
3 ENTER↑ 2 ENTER↑ 7	7	
÷	0.28571428571429	Result of 2/7
y^x	1.36873810664221	Result of 3 ^{0.28571428571429}
4 ENTER↑ 4 ENTER↑ 9	9	
÷	0.44444444444444	Result of 4/9

Press	Display	Comments
y^x	1.85174942457457	Result of $4^{0.444444444444444}$
+	3.22048753121678	Result of $1.36873810664221 + 1.85174942457457$
7 ENTER↑ 1 ENTER↑ 4	4	
÷	0.25	Result of $1/4$
y^x	1.62657656169779	Result of $7^{0.25}$
3 ENTER↑ 5	5	First do the exponent of the last term
÷	0.6	Result of $3/5$
8 $x\geq y$ y^x	3.4822022531845	Result of $8^{0.6}$
+	5.10877881488228	Result of $1.62657656169779 + 3.4822022531845$
÷	0.63038304219303	Result of $3.22048753121678 \div 5.10877881488228$

8. How to recognize and handle calculations that cannot be done using a 4-level stack?

Figuring out if a calculation can be done without overflowing the 4-level stack is a matter of counting. Sometimes the counting is easy, but sometimes it can be quite difficult. The simple counting technique explained below can even be used in the most complicated formulae.

Note that the following does not apply to the HP graphing RPN calculators (HP series 28, 48 and 49 and the HP 50g and Prime), because they have a virtually unlimited stack that is immune to overflow; also the WP 31S and WP 34S when operated in 8-level stack mode will in practice hardly ever overflow.

First we will show two impressive calculations that *can* be done using only the automatic registers of a 4-level stack.

Example XXV:*
$$\sqrt{\frac{8.33(4 - 5.2) \div [(8.33 - 7.46)0.32]}{4.3(3.15 - 2.75) - (1.71)(2.01)}}$$

Press	Display	Comments
4 ENTER↑	4	
5.2 −	−1.2	Result of $4 - 5.2$
8.33 ×	−9.996	Result of $(4 - 5.2) \times 8.33$
\square LAST x	8.33	Recall number that was in the display before the last operation
7.46 −	0.87	Result of $8.33 - 7.46$
0.32 ×	0.2784	Result of $(8.33 - 7.46) \times 0.32$
÷	−35.90517241	Result of $-9.996 \div 0.2784$, the numerator of the division
3.15 ENTER↑	3.15	
2.75 −	0.4	Result of $3.15 - 2.75$
4.3 ×	1.72	Result of $4.3 \times (3.15 - 2.75)$
1.71 ENTER↑	1.71	

Press	Display	Comments
2.01 \times	3.4371	Result of 1.71×2.01
$-$	-1.7171	Result of $1.72 - 3.4371$, the denominator of the division
\div	20.910356074	
\sqrt{x}	4.572784280	The final result

* This example premiered in the HP-11C Owner's Handbook (1981) and can be found in several later Manuals.

Example XXVI: *

$$\sqrt[5]{\left[\left(\left\{\left[\left(1 + 0.2 \left[\frac{350}{661.5}\right]^2\right)^{3.5} - 1\right] \times [1 - (6.875 \cdot 10^{-6}) \times 25500]^{-5.2656}\right\} + 1\right)^{0.286} - 1\right]}$$

Press	Display	Comments
350 ENTER↑	350	
661.5 \div	0.529100529	Result of $350/661.5$
x^2	0.279947370	Result of $[350/661.5]^2$
0.2 \times 1 $+$	1.055989474	Result of $[350/661.5]^2 \times 0.2 + 1$
3.5 y^x 1 $-$	0.210064617	Result of $([350/661.5]^2 \times 0.2 + 1)^{3.5} - 1$
1 ENTER↑	1.0	
6.875 E EX 6 CH S	6.875 -06	6.875×10^{-6}
ENTER↑	0.000006875	
25500 \times $-$	0.824687500	Result of $1 - 6.875 \times 10^{-6} \times 25,500$
5.2656 CH S y^x	2.759220911	Result of $[1 - 6.875 \times 10^{-6} \times 25,500]^{-5.2656}$
\times	0.579614684	Result of $0.210064617 \times 2.759220911$
1 $+$ 0.286 y^x	1.139687100	Result of $(0.579614684 + 1)^{0.286}$
1 $-$	0.139687100	
5 \times \sqrt{x}	0.835724536	The final result

* This example first appeared in the HP-25 Owner's Handbook (1975). It calculates the [Mach number](#) of an aeroplane flying with a calibrated airspeed (CAS) of 350 knots at a pressure altitude (PALT) of 25,500 feet.

Let's try to figure out why the two complicated expressions above (the second one with 6 levels of parentheses!) do not overflow the 4-level stack, while an innocent-looking expression such as $3[4 + 5(6 + 7)]$ goes awry when done in a left-to-right way.

There are a few simple principles you should remember in order to understand what goes on within the 4-level stack during a calculation:

- The stack can hold only 4 numbers, the number in the display and 3 additional numbers. These 4 numbers can either be newly keyed-in numbers, or intermediate results from the calculation or any combination thereof, but *the total may never exceed four!*
- So when the stack is full with 4 numbers you need to make room before you can key-in a new number.
- The only way to make room on the stack is by doing a two-number operation ($+$, $-$, \times , \div , y^x) with two numbers already present on the stack: two numbers are removed in the operation, and make room for the result and a free place. So a *two-number operation without entering any numbers makes room for one new number!*

- One-number operations leave the rest of the stack unchanged, and the same goes for two-number operations with one newly keyed-in number and one number already on the stack, so on a full stack you can do an unlimited series of such operations. But to make room you must do one or a few two-number operations with both numbers already on the stack.
- When entering new numbers remember that you need only three ENTERs to enter four numbers:
1 ENTER 2 ENTER 3 ENTER 4. In Classical RPN (but not in Entry RPN) machines when the stack is full only pressing ENTER (without keying in a number) is enough to overflow the stack. This is because in Classical RPN (not in Entry RPN) machines ENTER duplicates the number in the display (x) and pushes all numbers already in the stack upwards; in this action the number on the top of the stack is pushed 'over the top' and gets lost. A number subsequently keyed-in overwrites the number in the display and becomes the new x, but it is the ENTER that does the 'pushing' and so is responsible for the resulting overflow. See [Appendix A](#) for a thorough description of the stack.

With this knowledge it can easily be seen why you cannot do $3[4 + 5(6 + 7)]$ purely from left to right. Only after having entered 5 numbers (3, 4, 5, 6 and 7) can you do a two-number operation, and when entering 5 numbers (4 ENTERs) in a row the first number entered (the 3) will get lost (see [Example XIV](#)). So you could do this calculation starting with the 4 and working from there to the right, and finally doing the first operation (see [Example xv](#)).

When doing this calculation starting with the innermost pair of parentheses you can do the first two-number operation right after having entered the first two numbers, so immediately one stack place is freed. From there you do another two-number operation right after each new number keyed-in. So in this way you need only 2 of the four stack levels during the whole calculation. Look [here](#) for an illustration of the workings of the stack during this calculation.

Some calculations can be done both ways: $(e - 1)^{2/(\pi - \log \pi)}$

After having calculated $e - 1$ (which leaves one result on the stack) you would have to push three additional numbers (2, π and π) on the stack. So this calculation *can* be done from left to right on a 4-level stack. Of course you can also start from within the innermost set of parentheses, in this case the exponent, because this calculation could have been written thus: $(e - 1)^{(2/(\pi - \log \pi))}$. But that would complicate things a bit, because of the two SWAPs ($x \rightleftharpoons y$ or \blacktriangleright) you would have to do. See [Examples XXI](#) and [XXII](#).

Now, let's have a look at a calculation that is too much for the 4-level stack, no matter how you attack it. They're hard to find, but I found one (thanks to the [HP Forum](#) on the [Museum of HP Calculators](#)). Another one can be seen in the [Cheat Sheet under 7](#). In both calculations you will have to use **STO** and **RCL** to get the right answer on a 4-level stack RPN calculator:

$$[(3+1)(4+3) + (2+6)(4+6)] / [(2+3)(2+1) + (3+5)(4+2)]$$

By simply counting you can see why more than two intermediate results have to be remembered on the stack just before the last two-number operation (last pair of parentheses), thus overflowing the 4-level stack. In the following (•) represents one intermediate result on the stack.

We'll start on the left: after having calculated $(3+1)(4+3)$ there is 1 result on the stack:

(•)+(2+6)(4+6) / ...

Doing (2+6) after this takes 3 stack levels in total (the two new numbers plus the previous result waiting on the stack), and after this addition we have 2 results waiting on the stack:

(•)+(•)(4+6) / ...

Now we can do (4+6), and while doing this operation all 4 stack levels will be in use; after this we can finish the numerator of the division, leaving one result on the stack:

(•) / ...

Because the denominator of the division has the same structure as the numerator, it can easily be seen that at some point in doing the denominator, we'll have the following situation:

(•) / (•)+(•)(4+2), and finishing that calculation would require 5 stack levels, which is too much for a 4-level stack calculator: Q.E.D!

By counting in this way you can predict when 4 stack levels will not suffice. If you follow the rule of always doing calculations from the innermost pair of parentheses outwards you will hardly ever encounter such calculations in practice.

But if you do encounter one or if you're not sure if the 4 levels of your stack will suffice, then using **STO** and **RCL** is easy. In order to save one keystroke (**xzy**) if there is a non-commutative operation ($-$, \div , y^x) somewhere in the middle of the calculation, start with the part after this operation (in this case the denominator), **STO**re the intermediate result, and when the part before this operation (the numerator) is done, **RCL** (recall) the intermediate result and press the middle operation key:

Example XXVII:
$$\frac{(3+1)(4+3) + (2+6)(4+6)}{(2+3)(2+1) + (3+5)(4+2)}$$

Press	Display	Comments
2 ENTER 3 +	5	Start with the denominator if you have to use STO and RCL
2 ENTER 1 +	3	
×	15	
3 ENTER 5 +	8	The result of $(2 + 3) \times (2 + 1)$
4 ENTER 2 +	6	
×	48	
+	63	The denominator: $15 + 48$
STO	63	(or STO n *) Store in a non-stack storage location
3 ENTER 1 +	4	Now we continue with the numerator of the division
4 ENTER 3 +	7	
×	28	
2 ENTER 6 +	8	The result of $(3 + 1) \times (4 + 3)$
4 ENTER 6 +	10	
×	80	
+	108	The numerator: $28 + 80$
RCL	63	(or RCL n) Recall the denominator
÷	1.714285714	Final result of $108/63$

* The HP-35 had only one memory location apart from the stack; later models need a register number after **STO** and **RCL**.

The important lesson from this example is: remember that *at most **two** intermediate results can be kept on the stack if you still have to enter a new pair of parentheses with a two-number calculation in it, in other words: when you still have to press **ENTER** at least once!*

Appendices

- A. The Stack
- B. **LAST x**
- C. Percentage calculations

- D. [RPN keeps *you* in control](#)
- E. [Order of precedence of operators](#)
- F. [Things HP did \(and does\) not tell in the Manuals of some of their RPN-calculators](#)
- G. [RPN Variants](#)

Appendix A. The Stack

The classical 4-level automatic memory stack consists of four storage locations or registers (called X, Y, Z and T) that can best be seen as arranged one on top of the other, T being the top register, and X the bottom register. The display always shows the number in the X register, unless the calculator is in programming mode.

When you key-in a number, it goes into the X register, and so is displayed. Also the result of the last calculation will be stored into the X register, and so will be visible. Results of previous calculations are automatically stored into the Y, Z and T registers in a *last-in-first-out* manner.

T	0.000	The X register is always displayed
Z	0.000	
Y	0.000	
X	0.000	

In some models (HP-42S, HP 33s, HP 35s) also the contents of the Y register is shown in the display, above the contents of the X register, which can be handy in case of non-commutative operations ($-$, \div , y^x) to get the order of the two numbers right.

In the HP graphing calculators (HP series 28, 48 and 49, HP 50g and Prime) three or more stack levels are visible in the display.

The HP graphing calculators have much more than four stack levels, *numbered* from 1 upwards rather than using letters. In all except the Prime the number of stack levels is only limited by the amount of memory; the Prime has a fixed number of 128 stack levels. In this way the chance of stack overflow (loss of the number on the top of the stack) is virtually zero, or in case of the Prime greatly reduced. Another difference is the behaviour of the ENTER key, called '[Entry RPN](#)'. Even so most of what follows also applies to them. *Just think of the X register as the lowest number in the display: either the command/entry line or stack level 1.* For details on the differences see [Appendix G. RPN Variants](#).

In the examples below the stack is shown to be filled initially with the numbers 1.0 to 4.0 as the results of previous operations.

One-number calculations

T	4.000		4.000
Z	3.000		3.000
Y	2.000		2.000
X	1.000	e^x	2.718

One-number calculations only act on the number in the X register, and the result overwrites this number. In the example above by pressing the e^x key 1.0 is replaced by the (rounded) result of $e^{1.0}$. The other stack registers remain the same.

T	4.000		3.000		3.000
Z	3.000		2.000		2.000
Y	2.000		2.718		2.718
X	2.718	169	169.000	\sqrt{x}	13.000

When you key-in a new number, e.g. 169, it always goes into the X register, and so is displayed. The numbers in the other stack registers are automatically ‘pushed up’ one level to make room for the new number; *the number previously in the T register (4.000) is lost*. This ‘pushing up’ of the numbers in the stack when a new number is keyed-in is also called *stack lift*.

Now when you press another one-number function key, e.g. \sqrt{x} , again the number in the X register is replaced by the result of the function and the other stack registers remain the same.

So remember:

- Pressing a one-number function key only changes the number in the X register; the other stack registers (Y, Z, T) are unaffected.
- When you subsequently key-in a new number this appears in the X register; the result of the previous operation is ‘pushed up’ the stack and thus preserved

Two-number calculations

T		3.000		2.000		2.000		2.000
Z		2.000		2.718		2.000		2.000
Y		2.718		13.000		2.718		2.718
X	\sqrt{x}	13.000	12	12.000	+	25.000	%	0.679

In the figure above the number in the display (13.000) is the result of a previous operation. Hence if you now key-in another number, e.g. 12, it is recognized as a new number so the numbers in the stack are automatically ‘pushed up’ one level to make room for the new number; and the number previously in the T register (3.000) is lost.

Two-number functions always act on X and Y, and the result replaces the number in X. For example if you press $+$ then X is added to Y, and the result (25.0 in the above example) is stored in X. Except for percentage functions, in two-number functions the other numbers in the stack are ‘popped down’ one level (also called *stack drop*) and the contents of T (2.0) is copied to Z.

As you can see in the figure in case of *percentage functions* (% , $\Delta\%$ and %T) there is no stack drop: the number in the Y register is preserved and can be used in further operations (see [Example VIII](#) and [Appendix C](#) for examples of calculations with percentage functions). The percentage functions in the HP graphing calculators do not have this useful feature; there you have to press ENTER twice after the base number to simulate this behaviour.

The above shown two-number functions were done respectively with one newly keyed-in number and one number already on the stack, and with two numbers already on the stack. When you want to do a two-number function with *two newly keyed-in numbers* you need to use ENTER to separate the two numbers. Apart from this ‘number entry termination’ function, in a Classical RPN machine pressing ENTER has three other immediate effects, as shown in the next figure:

--	--	--	--	--	--

T	4.000		3.000		2.000		2.000		2.000
Z	3.000		2.000		1.000		1.000		2.000
Y	2.000		1.000		27.000		27.000		1.000
X	1.000	[27]	27.000	ENTER↑	27.000	[38]	38.000	+	65.000

- it duplicates the contents of X into Y, and thereby

- it causes a stack lift

- it then temporarily disables the stack lift, so when the next number is keyed-in it overwrites the number in X and the rest of the stack remains unchanged

This duplicating property of ENTER is the basis for a peculiar way to square numbers, e.g.: [7] ENTER↑ × with result: 49.000 . On the HP-35 this was the standard way to square a number, as there was no key for x^2 !

Pressing ENTER to square a number uses one extra stack level, so if your calculator has a separate x^2 key it is better to use that.

It is obvious that only two of the four numbers previously on the stack (1.0 and 2.0) are preserved when a two-number function with two newly keyed-in numbers is done; the numbers 3.0 and 4.0 previously on the stack are lost.

ENTER should only be used to separate two numbers keyed-in *sequentially* without another function in-between. Using ENTER twice (after each number keyed-in) in a Classical RPN calculator gives a false result (without warning!). This is shown in the next example:

T	4.000		3.000		2.000		2.000		1.000		1.000
Z	3.000		2.000		1.000		1.000	Wrong!	27.000		1.000
Y	2.000		1.000		27.000		27.000	↓	38.000		27.000
X	1.000	[27]	27.000	ENTER↑	27.000	[38]	38.000	ENTER↑	38.000	+	76.000 ← Wrong answer!

The answer is wrong because we calculated the result of $38 + 38$ and *not* of $27 + 38$.

In the recent models that use 'Entry RPN' (the HP 20b and 30b and in the graphing calculators of (HP series 28, 48 & 49, HP 50g and HP Prime) it is allowed to press ENTER↑ (called INPUT on the 20b and 30b) after the second number, but it is not necessary.

Remember:

- ENTER *normally should only be used to separate two numbers keyed-in sequentially (without a function key in-between)*
- On the classical 4-level stack by pressing ENTER↑ *once the results of only **two** previous operations will be preserved.*



Nested parentheses and stack overflow

When you start calculations with nested parentheses at the innermost set of parentheses the stack will not easily overflow.

E.g.: $3 \times [4 + (5 \times (6 + 7))]$

T	0		0		0		0		0		0		0		0		0		0		0
Z	0		0		0		0		0		0		0		0		0		0		0
Y	0		0		6		6		0		13		0		65		0		69		0
X	0	[6]	6	ENTER↑	6	[7]	7	+	13	[5]	5	×	65	[4]	4	+	69	[3]	3	×	207


But when you do these calculations from left to right on a 4-level stack then stack overflow is easily reached and you will get wrong answers (without warning).

E.g.: $3 \times [4 + (5 \times (6 + 7))]$ (left-to-right,  stands for )


T	0	0	0	0	0	0	3	3	4	4	4	4	4	4		
Z	0	0	0	0	3	3	4	4	5	5	4	4	4	4		
Y	0	0	3	3	4	4	5	5	6	6	5	4	4	4		
X	0	3	3	4	4	5	5	6	6	7	7	13	65	69	276	← Wrong answer!

The answer is wrong because the 3 is lost when the ENTER after the 6 is pressed (or when the 7 is pressed in Entry RPN models).


Stack manipulation











In Classical RPN calculators  is used *three* times to fill the stack with *four* numbers:




T	0.000	0.000	0.000	0.000	0.000	0.000	4.000	4.000							
Z	0.000	0.000	0.000	0.000	4.000	4.000	3.000	3.000							
Y	0.000	0.000	4.000	4.000	3.000	3.000	2.000	2.000							
X	0.000	4	4.000	ENTER↑	4.000	3	3.000	ENTER↑	3.000	2	2.000	ENTER↑	2.000	1	1.000

Important! When you would press  a *fourth* time after the fourth number then the first number (4.0) will be lost*:

T	4.000		3.000
Z	3.000		2.000
Y	2.000		1.000
X	1.000	ENTER↑	1.000

* Except in the above mentioned HP 20b/30b 'Entry RPN' calculators, where you *do* have to press  *four* times to fill the 4-level stack.

The key  is called *X exchange Y*. In some RPN calculators a similar function is called SWAP. The story of SWAP is slightly confusing, but don't get discouraged and read on. In the HP 20b/30b this is the  symbol on the  key, which functions identical to  when in RPN mode. In the HP 28 series SWAP is the shifted function of the  (Backspace) key. In graphing calculators that have a separate right arrow key  (HP 48 and 49 series and HP 50g) this key performs the SWAP function, but only *when there's no command line*; so sometimes you have to press ENTER before SWAP (). In the HP Prime the SWAP function  is under the comma key , but only *when the entry line is empty*; so here too sometimes you have to press ENTER before SWAP ().

These keys are used to exchange the contents of the lowest two stack registers (classically called X and Y), which can be very handy to get the order of numbers right before a *non-commutative* operation ,  or  when you do this operation *backwards*, i.e. from right to left in stead of from left to right (see [Examples XVI-XVIII](#)):

T	4.000	4.000
Z	3.000	3.000

Y	2.000		1.000
X	1.000	$x\geq y$	2.000

Stack inspection

Pressing $x\geq y$ twice leaves the stack unchanged, so in this way the contents of Y can be inspected:

T	4.000		4.000		4.000
Z	3.000		3.000		3.000
Y	2.000		1.000		2.000
X	1.000	$x\geq y$	2.000	$x\geq y$	1.000

$R\downarrow$ (called *roll down*) ‘rotates’ the contents of all four stack registers. Pressing this key four times lets you inspect the content of Y, Z, T and finally X again:

T	4.000		1.000		2.000		3.000		4.000
Z	3.000		4.000		1.000		2.000		3.000
Y	2.000		3.000		4.000		1.000		2.000
X	1.000	$R\downarrow$	2.000	$R\downarrow$	3.000	$R\downarrow$	4.000	$R\downarrow$	1.000

Of course in calculators with five or eight or more stack levels $R\downarrow$ ‘rotates’ the contents of all 5 or 8 or all occupied stack levels. In HP graphing RPN calculators similar but somewhat different functions are ROLL and ROLLD. Read the manuals for details.

Repeated calculations with a constant

When the numbers in the stack drop while doing a two-number operation the number in the T register remains constant and is copied to the next lower level. Because stack drop is also known as ‘popping the stack’ this is sometimes called *Top copy on pop*. Of course this only works when there is an upper limit to the stack. This property of RPN calculators with a limited stack can be used to do calculations with a constant.

Example XXVIII: If your firm has a projected growth rate in sales of 17% per year, when will the sales have doubled?
Solution: 17% per year makes an annual growth factor of 1.17. Let the current sales be 100.

Press	Display	Comments
1.17	1.17	The annual growth factor
$\text{ENTER}\uparrow$ $\text{ENTER}\uparrow$ $\text{ENTER}\uparrow$	1.17	Now the stack is filled with the constant *
100	100.00	Current sales
\times	117.00	Sales after 1 year
\times	136.89	Sales after 2 years
\times	160.16	Sales after 3 years

Press	Display	Comments
	187.39	Sales after 4 years
	219.24	Sales after 5 years

* In the Entry RPN calculators HP 20bHP 30b should be pressed *four* times to fill the 4-level stack with four numbers.

So within 5 years the sales will have doubled, given this constant growth rate.

Appendix B. LAST x

The first two HP calculators (the HP-35 and 80) did not have one, but all later Classical RPN calculators have a LAST X register, which automatically holds the contents of the X register just before the most recent operation. Pressing recalls this value into the X register. In the HP 20b and 30b the LAST x function is called ANS. In the WP 34S the LAST x function is performed by (=).

The WP 31S doesn't have a LAST x function but instead features , which restores the stack and all other registers to the state they were before the last command.

The HP graphical calculators have a LAST or LAST ARG or ANS function which serves a similar purpose, but recalls *all* arguments of the last operation to the stack.

These keys can be used to *correct entry mistakes* or to *reuse a number* in a calculation.

Use of to correct entry mistakes

One-number functions

If you pressed the wrong one-number function key, use to recall the number so you can press the correct function key.

E.g. In a long calculation you pressed by mistake, but wanted to press . You don't have to start all over again!

First delete the wrong result with or to prevent that it is saved on the stack.

Retrieve the number that was in the X register before the mistake and press the correct function key:

Two-number functions

Wrong function or *wrong second number*. Correct mistakes while keying in two-number functions with and the *inverse* of the function you used (+ vs. − or × vs. ÷). This brings back the state of the stack just before the mistake; then you can correct it. Use of or is only necessary if you're in the middle of a long calculation to prevent that the wrong number is put on the stack.

E.g. you want to calculate 65×32

You can make *three* kinds of mistakes:

Mistake

65 32 (wrong function)

65 332 (wrong second number)

665 32 (wrong first number)

Correction

()

()

()

32

() 65

Use of to reuse a number

There are two situations where this can be handy:



1. In a longer calculation where the same number appears twice with only one operation in-between: $(567.89012 + 1.23456789) \div 1.23456789$



567.89012  1.23456789     Answer: **460.9910014**

2. In a repeating short calculation as a semi-automatic constant.

E.g.: the price of an item is € 123.25; what would be the cost of 7, 13 or 17 of these items?

Enter the constant *last*:

7  123.25  Cost: **862.75**

13    Cost: **1,602.25**


17    Cost: **2,095.25**



Appendix C. Percentage calculations

Percentage functions (% , Δ% and %T) are a bit special because in many RPN calculators the first (base) number is preserved during the operation and can be used in further calculations. In some (notably the RPN graphing) calculators the base number is not preserved; there you'll have to press ENTER *two* times between the base and the percentage number to get the same effect. Also in the RPN graphing calculators the designations of these functions may differ: Δ% may be called %CH or %CHANGE and %T may be called %TOTAL.

Example XXIX: A car is listed for \$29,690. You deal a 7% discount and the sales tax is 9.5%. What will be the total purchase price?




Press	Display	Comments
29690	29,690	The first or <i>base</i> number (the list price)
	29,690.00	 separates the first from the second (percentage) number*
7	7	The percentage discount
	2,078.30	7% of \$29,690
	27,611.70	Net price before tax: \$29,690 minus 7% †
9.5	9.5	The sales tax percentage
	2,623.11	9.5% of \$27,611.70
	30,234.81	Total price: \$27,611.70 plus 9.5%

* In RPN graphing calculators press  *twice*!

† In RPN graphing calculators here you'll have to press  to duplicate this result before keying-in the percentage and pressing 

Example xxx: Δ% calculates the percent change between two numbers, negative if the second is lower than the first. Because the base number is saved you can easily make multiple comparisons.

E.g.: Yesterday the stock price of a share fell from 61¼ to 53½. What is the percent change? And what if the price rose to 65 or 70?

Press	Display	Comments
61.75	61.75	The first or <i>base</i> number
	61.75	 separates the first from the second number*
53.50	53.50	Key-in the second number
	-13.36	More than 13% decrease

Press	Display	Comments
CL x	0.00	Clear the X-register to make a new comparison
65	65.00	What if the stock rose to 65?
Δ%	5.26	Loosely 5% increase
CL x	0.00	Again clear the X-register
70	70.00	What if it rose to 70?
Δ%	13.36	Interesting!

* In RPN graphing calculators press **ENTER↑** twice!

Example XXXI: Goods are purchased for a wholesale price of \$22 and sold for a retail price of \$29.99. What is the *markup* (price difference as a percentage of cost price (wholesale price)) and what is the *margin* (price difference as a percentage of selling price (retail price))?

Press	Display	Comments
22	22.	The first or <i>base</i> number: the cost (wholesale price)
ENTER↑	22.00	ENTER↑ separates the first from the second number*
29.99	29.99	Key-in the retail price
Δ%	36.32	The <i>markup</i> is over 36%
LAST x	29.99	Recall the retail price; now this is the base number
22	22.0	Key-in the wholesale price as the second number
Δ%	-26.64	The change is negative
CH S	26.64	Change the sign: the <i>profit margin</i> is less than 27%

* In RPN graphing calculators press **ENTER↑** twice!

Example XXXII: %T calculates what percentage of a total a certain part of that total is. E.g. in a population there are 356 men, 401 women and 289 children. What percentage of the total population do the three numbers account for?

Press	Display	Comments
356	356	The number of men
ENTER↑ 401	401	The number of women
+	757	Add the two numbers
289 +	1,046	The number of children added; so this is the total
356	356	Again type the number of men
%T	34	Men: 34%
CL x 401	401	Clear the display and give the number of women
%T	38	Women: 38%
CL x 289	289	Clear the display and give the number of children
%T	28	Children: 28%

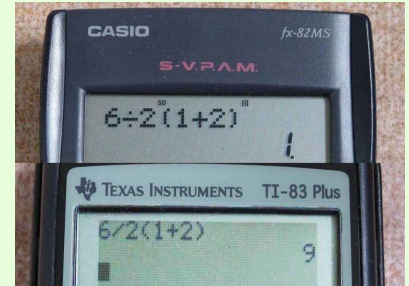
The behaviour of the stack in percentage functions is described [here](#).

Appendix D. RPN keeps *you* in control

If you let your 'algebraic' calculator or spreadsheet computer program decide over the operator precedence of a calculation you might not get the right answer. The most notorious example is a calculation like $6 : 2 \times (1 + 2)$, which is interpreted differently by Casio and TI (see [Appendix E](#) to find out which one is right):

Another example is the treatment of the unary minus as having a higher precedence than exponentiation by Microsoft's spreadsheet application Excel, which contends that $-3^2 = (-3)^2 = 9$ whereas the correct answer is: $-3^2 = -(3^2) = -9$

Also Excel evaluates 4^{3^2} (written as $4^{\wedge}3^{\wedge}2$) as $(4^3)^2 = 64^2 = 4096$. The usual interpretation of 4^{3^2} is to work from the top down, so there are implied parentheses in the exponent: $4^{(3^2)} = 4^9 = 262,144$



Because in RPN the order of operations is not decided by the calculator but by the user it is wise to brush-up on this important issue (see the next Appendix).

Appendix E. Order of precedence of operators

An RPN calculator won't do it for you, so it is important that you have the simple rules of mathematical operator precedence in your head.

The [standard order of operations](#) is as follows:

1. Parentheses (brackets)
2. Exponents and roots
3. Multiplication and division
4. Addition and subtraction

Mnemonic: **PEMA!**

Operations that are on the same line in the above list, such as multiplication and division, have *equal* precedence. Mixed operations with equal precedence (e.g. a series of multiplications and divisions) are evaluated *from left to right*.

What is the correct way to calculate $6 : 2 \times (1 + 2)$?

Because multiplication and division have the same operator precedence, when next to each other they should be evaluated from left-to-right:

$$6 : 2 \times (1 + 2) = 3 \times (1 + 2) = 3 \times 3 = 9$$

It may help to remember that division by 2 is the same as multiplication by $\frac{1}{2}$ because then it is easier to do the same calculation backwards:

$$6 : 2 \times (1 + 2) = 6 \times \frac{1}{2} \times 3 = 6 \times 1.5 = 9$$

Another correct way is thus:

$$6 : 2 \times (1 + 2) = (6 : 2) \times (1 + 2) = (6 : 2) \times 3 = 3 \times 3 = 9$$

It is *wrong* to do it like this:

$$6 : 2 \times (1 + 2) = 6 : (2 \times (1 + 2)) = 6 : (2 \times 3) = 6 : 6 = 1 \text{ (wrong!)}$$

Using an RPN calculator this calculation can be done in several ways:

6 [ENTER] 2 [÷] 1 [ENTER] 2 [+] [×]

or

1 [ENTER] 2 [+] 2 [1/x] [×] 6 [×]

or

1 **ENTER↑** 2 **+** 2 **ENTER↑** 6 **x↯y** **÷** **×**

or

1 **ENTER↑** 2 **+** 6 **ENTER↑** 2 **÷** **×**

All give the same result: 9

How to apply the *unary minus* operator?

$-3^2 = -(3^2)$ = should be calculated on an RPN calculator like so:

3 **ENTER↑** 2 **y^x** **CH S**

Result: -9

When negating a number it is important to *always* press **CH S** *after* keying-in the number and *before* pressing **ENTER↑**. For a discussion of why this is important read [this article](#).

What is the correct way to calculate 4^{3^2} ?

Using an RPN calculator this calculation can be done in two ways:

4 **ENTER↑** 3 **ENTER↑** 2 **y^x** **y^x**

or

3 **ENTER↑** 2 **y^x** 4 **x↯y** **y^x**

Both give the same result: 262,144 (see Examples XVIII and XIX)

Appendix F. Things HP did (and does) not tell in the Manuals of some of their RPN-calculators

Left-to-right or from within the innermost parentheses?

The HP-35 Operating Manual (1972) concludes with an RPN algorithm and flow chart ‘adequate for most practical problems’. Because it says to key-in the numbers ‘in the order in which they occur’ (that is from left to right) this algorithm fails in some quite simple calculations with nested parentheses (e.g. see [Example XIV](#) above).

The same algorithm and a similar flow chart were given in the HP-45 Owner’s Handbook (1973). Later HP published a brochure titled ‘ENTER↑ vs. =’, which was devoted solely to the basics of RPN calculators as opposed to Algebraic notation calculators. This brochure contained a simplified version of the flow chart, having the same flaw. It was the last time HP used this flow chart (as far as I know), although the algorithm it depicts is perfectly valid in the later RPL calculators with (virtually) unlimited stack length.

In the manuals of all other early RPN Calculators (HP-80, 70, 65, 55) a similar advise is given: ‘Almost all problems can be solved (...) from left-to-right.’ But it was left as an exercise to the user to figure out when a problem could not be solved this way, and how then to proceed.

Only several calculator models later, in the Owner’s Handbook of the HP-25 (1975) can we read ‘It is best to start *every* problem at its innermost number or parentheses and working outward.’ Since then a similar advice is given in the Handbook of every next model. But even at the present day the HP-12C and HP-12C Platinum User’s Guides give this advise only in an appendix.

Use of SWAP

Until 1983 the use of **x↯y** in nested calculations when you have to do a *non-commutative* operation ($-$, \div , y^x) ‘backwards’ was only hinted at in every manual. Starting with the HP-41CX Manual (1983) this point is worked out more explicitly. But even in 2014 the current versions of the HP-12C and HP-12C Platinum User’s Guides do not mention this important point, not even in the Appendix about the stack. In the later graphical calculators (HP-49 series and HP-50g) the use of **►** for the SWAP function is hardly documented in the Manuals and the function is not printed on the keyboard.

Stack Overflow

The gravest omission in the User's documentation of every single HP classical 4-level stack RPN calculator up to the present day is the failure to mention the possibility of *stack overflow*. Nowhere is it mentioned explicitly that sometimes wrong answers are given *without warning*.

And conflicting information is given about the number of intermediate results that are preserved on the 4-level stack in various manuals. The HP-55 Owner's Handbook (1975) states that 'up to *four* intermediate results are stored in the stack' (italics are mine); using the same example the HP-31 Owner's Handbook (1978) says 'the automatic memory stack stores up to *three* intermediate results until you need them, and then inserts them into your calculation'. Not even in the newest HP Manual is it admitted that quite often only *two* intermediate results are preserved, specifically when you have to use **ENTER↑** once again, e.g. to put two new numbers on the stack.

In the opinion of the author of this RPN Tutorial this failure by HP to mention the obvious limitation of the 4-level stack and its consequences *explicitly* has done the acceptance of RPN more harm than good.

Appendix G. RPN variants

Often the two main variants of RPN are referred to as 'Classical RPN' and '**RPL**'. This is a bit confusing because **RPL** is actually the name of the programming language used in the HP graphing calculators of series 28, 48 & 49 and also in the HP 50g. The HP Prime has another programming language but uses a similar variant of RPN.

Purely looking at the workings of the stack of RPN calculators (and not at their programming capabilities) a few RPN variants can be discerned. Both the bottom and the top of the stack may vary.

A. Bottom of the stack:

1. Classical RPN
2. Entry RPN

B. Top of the stack:

1. Small stack (4-, 5- or 8-level) with 'top copy on pop'
2. Very large stack (>100 levels), no 'top copy on pop'

Four types of RPN


All four possible combinations of the 2×2 possibilities of RPN variants actually do exist, although not always in hardware:

- a. The combination of *Classical RPN* with a *small stack* and 'top copy on pop' is well known. Besides the 4-level stack of all classical HP RPN calculators (all HP RPN calculators except those listed below after β. and γ.) and their emulations there is the 5-level stack of the Heathkit OC-1401, and the 8-level mode of the **WP 31S** and **WP 34S**.
- β. All HP RPN graphing calculators (series 28, 48 & 49, the HP 50g and Prime) have *Entry RPN* and an unlimited or *very large* stack (128 stack levels in the Prime).
- γ. The HP 20b and HP 30b have *Entry RPN* and a *4-level stack* with 'top copy on pop'.
- δ. The **Calculator** application in Mac OS X in RPN-mode combines *Classical RPN* behaviour of the ENTER key with an *unlimited stack*.

The Bottom of the stack: Classical RPN versus Entry RPN

The differences at the *bottom* of the stack need not bother you as long as you stick to the rules given in the 'Cheat Sheet'. Especially observe the rule to use **ENTER↑** *only* to separate two numbers that have to be keyed in immediately one after the other. When you can't see the stack do not uncritically rely on **ENTER↑** to *duplicate* a number on the stack and do *not* use **ENTER↑** to 'enter' a number into the stack.

There are two important exceptions to this latter rule worthwhile to remember, which only apply to the HP graphing calculators (so it's easy to use them because you can see the stack):

- In most HP graphing calculators SWAP only works when the command line / entry line is *empty*, so often it will be necessary to press Enter before ► or ✕ () when you want to exchange the lowest two stack levels.

- % functions in HP graphing calculators do not automatically preserve the base (first) number like in other HP RPN calculators, so when you want to use that number in further calculations (e.g. to add a percentage) you will have to press Enter *twice* between the base and the percent to duplicate the base number.

If you want to know what's happening on the stack in Classical RPN and Entry RPN then the following is some explanation.

$$\begin{array}{r} 3 \\ 4 \\ \hline 7 \end{array} +$$

Classical RPN: 3 ENTER↑ 4 + Answer: 7
 3 ENTER↑ 4 ENTER↑ + Answer: 8 (wrong!)
 Entry RPN: 3 ENTER↑ 4 + Answer: 7
 3 ENTER↑ 4 ENTER↑ + Answer: 7

So Entry RPN is more forgiving, because you cannot do it wrong!

[Here](#) in Appendix A can you see how the wrong answer in Classical RPN calculators is produced.

In practice the difference between Classical RPN and Entry RPN is of no importance as long as you stick to the rule to use ENTER↑ *only* to separate two numbers that have to be keyed in immediately one after the other and *not* to 'enter' a number into the stack.

The mechanism of this difference on the stack can easily be seen in the following figures.

The first key sequence above (plus an extra ENTER↑) on a Classical RPN calculator:

T	0.000	0.000	0.000	0.000	0.000	0.000					
Z	0.000	0.000	0.000	0.000	0.000	0.000					
Y	0.000	0.000	3.000	3.000	0.000	7.000					
X	0.000	3	3.000	ENTER↑	3.000	4	4.000	+	7.000	ENTER↑	7.000

As you can see in the above figure in Classical RPN:

- ENTER↑ *always* duplicates X into Y (after number entry and after pressing a function key).
- But when followed by a next number entry the duplication is neutralized because the number in the X register is overwritten! This function of the ENTER key is also known as 'temporary stack lift disable'.

Remember: in Classical RPN calculators if ENTER↑ is not followed by a number you always have the number twice on the stack.

Now the same key sequence on the HP Prime, which has Entry RPN:

...																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																				
-----	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

In Entry RPN pressing ENTER sometimes moves and sometimes duplicates:

- In graphing calculators it is easy: if the command/entry line is empty ENTER duplicates else it copies.

- In other Entry RPN calculators remember: *if ENTER is pressed after ENTER or after a function key then you have the number twice on the stack, even if followed by a number entry.*
- Duplication in Entry RPN is never neutralized!

The Top of the stack: Small versus very large/‘unlimited’ stack. ‘Top copy on pop’

The height of the stack is important to know for all users because a small stack always leaves open the possibility of stack overflow, while in manual operation a very large stack is virtually immune to this error. Items 5. 6. & 7. of the ‘Cheat Sheet’ give hints on how to prevent stack overflow in calculators with a small stack.

The other difference at the *top* of the stack that might be important is the presence or absence of *top copy on pop*, which is only present in small stack models. See [Example XXVIII](#). In calculators with an very large stack you can somewhat emulate this behaviour by pressing Enter several times to replicate the constant number into the stack.

Which type of RPN does my calculator have?

To determine if your RPN calculator has Classical RPN or Entry RPN behaviour of the bottom of the stack do the following test. First bring the stack in a consistent pre-condition by pressing:

0 **ENTER↑** 0 **ENTER↑**

Then press:

5 **ENTER↑** **×**

If the result is **25** then your calculator has Classical RPN behaviour of the ENTER key.

If the result is **0** then your calculator has Entry RPN behaviour of the ENTER key.

The stack *size* should be evident from the documentation. But if you don't have that information you could do the following test.

First press:

0 **ENTER↑** 0 **ENTER↑**

Then press:

1 **ENTER↑** 2 **ENTER↑** 3 **ENTER↑** 4 **ENTER↑** 5 **ENTER↑**
6 **ENTER↑** 7 **ENTER↑** 8 **ENTER↑** 9 **ENTER↑** (don't forget the last ENTER!)

Then press 8 times **+**

- If the result is **45** then the calculator has more than eight stack levels (possibly an unlimited number) and Entry RPN (β -type).
- If the result is **53** then the calculator has more than eight stack levels (possibly an unlimited number) and Classical RPN (δ -type).
- If the result is **54** then the calculator has an 8-level stack and Classical RPN (α -type).
- If the result is **60** then the calculator has a 4-level stack and Entry RPN (γ -type).
- If the result is **63** then the calculator has a 5-level stack and Classical RPN (α -type).
- If the result is **68** then the calculator has a 4-level stack and Classical RPN (α -type).
- If the result is **81** then you have a calculator of the HP 9100 or 9800 series.

The results for the small stack sizes assume that on stack drop the top level copies its contents to the next lower level (“Top copy on pop”).



The HP RPN graphing calculators

How different is RPN in the HP graphing calculators? Apart from their virtually unlimited stack the only three important dissimilarities are:

1. A slightly different behaviour of the ENTER key: in these calculators ENTER may (but doesn't *need to*) be used *after* each number keyed-in to get it on the stack, so it is allowed to be used like an Enter-key on a computer keyboard.

This *postfix* use of the ENTER key is also allowed in the HP 20b and HP 30b (that aren't graphing calculators and *do* have a 4-level stack; in these models the ENTER key is labelled INPUT). In an article in *HP Solve* ([# 27 p. 42](#)) Richard Nelson calls this possibly *postfix* use of the ENTER / INPUT key ‘Entry RPN’, as opposed to the compulsory *prefix* use of ENTER in ‘Classical RPN’.

In 'Classical RPN' ENTER is normally only allowed to be used to separate two numbers keyed-in without pressing an operation key in-between. So it is used like the **E EX** key ('Enter Exponent'), which also is pressed *before* keying-in the exponent. This almost exclusive *prefix* use of the Classical RPN ENTER key could be expressed more clearly if it were labelled **E NXT** or **E NXT #** or **E NN** (for 'Enter Next Number') or simply **ENTER:**.

2. The use of an unnumbered *command line* (called *entry line* in the HP Prime) while keying in a new number, which allows to edit the number conveniently. When you press ENTER or a function key the number or the result is placed in stack level 1, and the command line disappears. The command line can be regarded as a temporary 'stack level 0'. So the lowest stack level (called X in classical RPN) alternates between the command line and level 1 of the stack. In the examples below think of the number in the X register as being on the lowest stack level (either on the command line, 'level 0', or on stack level 1). In **RPL** calculators the command line has more advanced uses that are beyond the scope of this tutorial.
3. SWAP is not always available as a primary key function. When the command line is *not* visible a SWAP of level 1 and 2 of the stack is done by just pressing ► on the calculators that have this key (HP 48 and 49 series and HP 50g) or pressing ⌘ (the comma key ) on the HP Prime. When the command line is visible just press ENTER to copy the contents of the command line to stack level 1 before pressing ► or ⌘. In the HP series 48 calculators when the command line is visible you may press the left shift key before pressing the ► key to SWAP what is on the command line with level 1 of the stack. In the HP 28 series SWAP is the shifted function of the  (Backspace) key.

A special case are the new WP 31S and WP 34S calculators, that are not produced by HP but are community-created products based on reprogrammed and relabeled HP 20b or HP 30b hardware. These calculators have the optional use of an 8-level stack and have classical behaviour of the ENTER key. The additional stack levels are called A, B, C and D.

A very special case are the first RPN calculators produced by HP, the tabletop models of series 9100 and 9800. They had a 3-level stack and would qualify als Classical RPN according to the first test. But the RPN was quite different. More on this on the pages of the [Museum of HP Calculators](#). It is interesting to note that these early calculators already had a kind of 'pseudo-Entry line': all input went into the lowest *X keyboard* register; pressing the ↑ (Enter) key copied this value to the *Y accumulator* register. The results of two-number operations went into this same *accumulator*, but results of one-number functions went into the *X keyboard* register. Intermediate results were pushed automatically into the *Z temporary* register, but there was no automatic stack drop. Fortunately the contents of all three registers were visible in the display.