INT

A Symbolic Integration Library for the HP48 with ALG48 Version 0.1

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1 Acknowledgements, copyright & disclaimer of warranty

2 Overview

The `INT` library provides commands for computing the derivative and indefinite integral (antiderivative) of algebraic expressions. The integration command in `INT` was designed as a heuristic front-end for a more general integration command (not yet implemented). The goal of this front-end is to "quickly" solve simple integrals or parts of integral, before a more general (and slower) algorithm is applied. At this stage, the integration command in `INT` can integrate rational combinations of exponential, hyberbolic, or trigonometric functions of the integration variable, as well as other kinds of relatively simple expressions (see Section 5 for details and examples).

The `INT` library works in conjunction with the `ALG48` library © by the same authors, and in the future, when the code is stable enough, it will probably be integrated directly into the main `ALG48` library.

3 Installation

`INT` takes approximately 7Kb of memory and works in both `HP48`48gx and `HP48`48sx. `INT` uses some of the internal routines in `ALG48`, and `ALG48` needs to be installed for `INT` to work. See the `ALG48` documentation for information on installing `ALG48`.

Warning: Only use this library with the version of `ALG48` with which it was distributed. Do not use it with an older version of `ALG48` or with the `RSIM` library. If you do it will probably crash you calculator.

Due to the special optimization features used in `ALG48`, not all storage combinations are allowed. The following ones are possible:

- In `SX` there are no restrictions
- In `GX` if `ALG48` is in port 0 or port 1 then `INT` can be stored in any port.
- In `GX` if `ALG48` is stored in port 2 (or higher) then `INT` must be stored in the same port, or port 0.

Installing `ALG48` and `INT` in the same port seems the most natural choice and is recommended.

`INT` is an auto-attaching library (library number 913). To install it on your `HP48` download the file `int.lib` onto your calculator (in *binary* mode), put the content of the created variable on the stack, store it the port of your choice (e.g., `'INT.LIB' DUP RCL SWAP PURGE 0 STO`) and power-cycle the calculator.

4 Commands

The `INT` library provides the following two commands:

`INTGR` – symbolic indefinite integral;

`DERIV` – symbolic derivate.

Both commands takes two arguments: the symbolic to integrate or derive on Stack Level 2, and the integration or derivation variable on Stack Level 1. The result of the commands is simplified and expanded along the non-rational functions of the integration or derivation variable, with the other variables treated as parameters. E.g.,

$$x^2\cos(ax) \quad \texttt{X INTGR} \Rightarrow \frac{x^2a^2-2}{a^3}\sin(ax)+\frac{2x}{a^2}\cos(ax) \quad \texttt{X DERIV} \Rightarrow x^2\cos(ax).$$

Even though it is not explicit in the output, like any indefinite integral, the integral returned by `INTGR` is defined up to an additive constant. That is, the general solution for the indefinite integral is the output of `INTGR` plus an arbitrary constant.

Section 5 describes in details the kind of expressions `INTGR` can integrate. Both `INTGR` and `DERIV` recognize the following operations and functions:

- basic operations: `+ - * / ^ NEG INV SQ SQRT XROOT`
- exponential: `EXP LN EXPM LNP1`
- trigonometric: `SIN SIN COS TAN ASIN ACOS ATAN`
- hyperbolic: `SINH COSH TANH ASINH ACOSH ATANH`

All the trigonometric functions are integrated and derived assuming that the angles are given in radian, regardless of the calculator's angle mode. `INTGR` integrates what it can and returns the rest unsolved inside a `INT` function. For instance,

$$e^{-x}+e^{-x^2} \quad \texttt{X INTGR} \Rightarrow -e^{-x}+\mathrm{int}(e^{-x^2},x).$$

The command `DERIV` also recognizes the `INT` function and given the above right-hand-side expression to derive with respect to x it would return the original expression. The functions that `DERIV` does not recognize are returned inside a '$\delta x(.)$' expression. Evaluating such an expression applies the calculator derivation routine. E.g.,

$$\mathrm{abs}(x) \quad \texttt{X DERIV} \Rightarrow \delta x(\mathrm{abs}(x)) \quad \texttt{EVAL} \Rightarrow \mathrm{sign}(x).$$

5 Scope of the Integration Command and Examples

At this stage, the integration command **INTGR** can solve the following kind of integrals. Below, x always denotes the integration variable.

1. Rational functions, in the same way as the command **RINT** in **ALG48**,

$$\frac{1}{x^2-2} \quad \texttt{x INTGR} \Rightarrow -\frac{1}{4}\sqrt{2}\cdot\ln(x+\sqrt{2})+\frac{1}{4}\sqrt{2}\cdot\ln(x-\sqrt{2})$$

2. Rational combinations of exponential and hyperbolic functions (exp, expm, cosh, sinh, and tanh),

$$\frac{e^{-2x}-1}{e^{-4x}+1} \quad \texttt{x INTGR} \Rightarrow \frac{1}{2}\mathrm{atan}(e^{2x})-\frac{1}{4}\ln(e^{4x}+1)$$

3. Rational combinations of trigonometric functions (sin, cos, and tan),

$$\frac{1}{\sin(x)+\cos(x)} \quad \texttt{x INTGR} \Rightarrow \frac{1}{2}\sqrt{2}\ln\left(\tan(\frac{1}{2}x)-1+\sqrt{2}\right)-\frac{1}{2}\sqrt{2}\ln\left(\tan(\frac{1}{2}x)-1-\sqrt{2}\right)$$

4. Simple non-rational functions (see list in Section 4) and their inverse (when the integral exists),

$$\mathrm{asin}(3x+2) \quad \texttt{x INTGR} \Rightarrow \frac{1}{3}\sqrt{-9x^2-12x-3}+\left(x+\frac{2}{3}\right)\mathrm{asin}(3x+2)$$

5. Expressions of the form $\sqrt{ax^2+bx+c}$ and their inverse,

$$\sqrt{8x^2+3x-1} \quad \texttt{x INTGR} \Rightarrow \left(\frac{1}{2}x+\frac{3}{32}\right)\sqrt{8x^2+3x-1}-\frac{41}{256}\sqrt{2}\ln\left(\sqrt{8x^2+3x-1}+2x\sqrt{2}+\frac{3}{8}\sqrt{2}\right)$$

6. Products of functions of the form $f(x)f'(x)$ or $f(g(x))g'(x)$,

$$\sin(x)\cos(x)^n \quad \texttt{x INTGR} \Rightarrow -\mathrm{inv}(n+1)\cos(x)^{n+1}$$

7. Ratio of functions of the form $f'(x)/f(x)$, or $g'(x)/f(g(x))$,

$$\frac{\cos(\ln(x))}{x\sqrt{\sin(\ln(x))}} \quad \texttt{x INTGR} \Rightarrow 2\sqrt{\sin(\ln(x))}$$

8. Expressions the form $p(x)f(x)$ where $p(x)$ is a polynomial in x and $f(x)$ is a function with a simple integral,

$$(x^2+2)\cosh(3x) \quad \texttt{x INTGR} \Rightarrow \left(\frac{1}{3}x^2+\frac{20}{27}\right)\sinh(3x)-\frac{2}{9}\cosh(3x)$$

9. Expressions of the form $p(x)\ln(q(x))$ where both $p(x)$ and $q(x)$ are polynomials in x,

$$x^2\ln(x^2+1) \quad \texttt{x INTGR} \Rightarrow -\frac{2}{3}\mathrm{atan}(x)+\frac{1}{3}x^3\ln(x^2+1)-\left(\frac{2}{9}x^3-\frac{2}{3}x\right)$$

10. Linear combinations of integrals above,

$$\frac{5\mathrm{atan}(3x+1)-2}{9x^2+6x+2} \quad \texttt{x INTGR} \Rightarrow \frac{5}{6}\mathrm{atan}(3x+1)^2-\frac{2}{3}\mathrm{atan}(3x+1).$$

6 Remarks

Here are a few additional things to note about `INT`'s commands.

- The derivation command in the `INT` library is not as complete as the calculator's command and only handles the functions listed in Section 4. It has, however, a couple of advantage over the calculator's command:
  - It handles fractions symbolically (i.e., without evaluating them);
  - The result is simplified;
  - It is slighlty faster than the calculator's command.
- As explained above, the results of `INTGR` and `DERIV` are simplified (as by an application of `RSIM` in `ALG48`). Sometimes however they can be much further simplified by the command `ASIM` in `ALG48`. E.g.,

$$\frac{1}{\sqrt{x}\sin(\sqrt{x})} \quad \texttt{x INTGR} \Rightarrow \; 2\ln(\tan(\frac{1}{2}\sqrt{x})) \quad \texttt{x DERIV} \Rightarrow \; \frac{\tan(\frac{1}{2}\sqrt{x}))^2+1}{2\sqrt{x}\tan(\frac{1}{2}\sqrt{x}))} \quad \texttt{ASIM} \Rightarrow \; \frac{1}{\sqrt{x}\sin(\sqrt{x})}$$

- Before reporting a bug about `INTGR`, remember that the indefinite integral of an expression is defined up to a constant and that it can generally be expressed in many different ways. If the `INTGR` command doesn't return the result you expect it is not necessarily a bug.

7 Contact

Gifts `:-)`, bug reports (see comment above), and constructive comments and suggestions can be sent to either one of the following addresses.

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